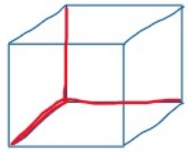
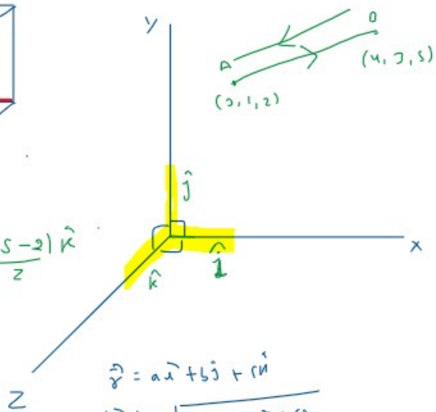
Vector Algebra

Wednesday, August 3, 2022 5:15 PM

Sc.





$\vec{BA} = -\hat{i} - 2\hat{j} - 3\hat{k}$

$= -(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{BA} = -\vec{AB}$

$\vec{AB} = (4-3)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k}$

$\vec{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let $\vec{a}$ = position vect

$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

$|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$

O (0,0,0) ⟶ P (2,3,1) ⟶ position vector

$\vec{OP} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$

Magnitude $= |\vec{r}| = \sqrt{2^2 + 1^2 + 1^2}$

$\vec{r} = 2\hat{i} + \hat{j} + \hat{k}$

$= \sqrt{6}$

Q.1 Gn $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$

find (i) $\vec{a} + \vec{b}$ = Resultant vect. $= 5\hat{i} + 0\hat{j} - 3\hat{k}$ = Let $\vec{c}$

(ii) $\vec{a} - \vec{b}$ $\quad |\vec{c}| = |\vec{a} + \vec{b}|$

(iii) $|\vec{a} + \vec{b}|$ (v) $\hat{a}$ $\quad |\vec{c}| = \sqrt{25 + 0 + 9}$

(iv) $|\vec{a} - \vec{b}|$ $\quad \hat{a} = \frac{3}{\sqrt{11}}\hat{i} + \frac{1}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k}$ $= \sqrt{34}$

* Unit vector $= \hat{r}$ = Unit vectr

Magnitude = 1

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{r} = 2\hat{i} - \hat{j} + \hat{k}$

$\hat{r} = ?$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}} = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k} = \hat{r}$

$|\hat{r}| = \sqrt{\frac{4}{6} + \frac{1}{6} + \frac{1}{6}} = \sqrt{\frac{6}{6}} = \sqrt{1} = 1$

Equal vector

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{b} = x\hat{i} - \hat{j} + y\hat{k}$

$x = ?$ $y = ?$

$\vec{a} = \vec{b}$

$x = 2$
$y = 1$

Ex $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ ⟶ 8 Magnitude

$|\vec{a}| = \sqrt{6}$

$\hat{a} = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

$8\hat{a} = \frac{16}{\sqrt{6}} - \frac{8}{\sqrt{6}}\hat{j} + \frac{8}{\sqrt{6}}\hat{k}$

Ma

4 kg = ₹200 /-

7 kg = ? ← 1 kg.

$\vec{a} \cdot \vec{b}$ = dot product

$\vec{a} \times \vec{b}$ = cross product

Δ Law

$\frac{\cdot}{\sqrt{6}} - \frac{\cdot}{\sqrt{6}}\hat{j} + \frac{\cdot}{\sqrt{6}}\hat{k}$

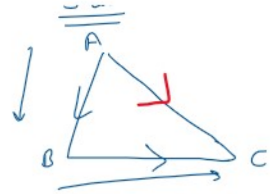
$$= \sqrt{\frac{256}{6} + \frac{64}{6} + \frac{64}{6}}$$

$$= \sqrt{\frac{384}{6}}$$

$$= \sqrt{64}$$

$$= \underline{\underline{8}}$$

64, 64, 128, 256, 384



$$\boxed{\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}}$$

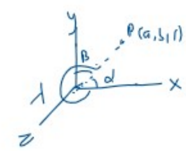
$$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

Direction Ratio's

Let $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

$a, b, c \rightarrow$ DR's



$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\lambda = 1}$$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

$l = \cos\alpha$, $\alpha = \cos^{-1} l$ (x-axis)

$m = \cos\beta$, $\beta = \cos^{-1} m$ (y-axis)

$n = \cos\lambda$, $\lambda = \cos^{-1} n$ (z-axis)

Direction Cosines

$$\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{a}{\sqrt{a^2+b^2+c^2}}\hat{i} + \frac{b}{\sqrt{a^2+b^2+c^2}}\hat{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\hat{k}$$

$\downarrow l$, $\downarrow m$, $\downarrow n$

DC's = l, m, n

$l = \frac{a}{\sqrt{\ }}$ $\quad m = \frac{b}{\ }$ $\quad n = \frac{c}{\ }$

Q. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$

find DR's, DC's

DR's $= \langle 2, 1, 1 \rangle$

DC's

$$\hat{a} = \frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

$l = \frac{2}{\sqrt{6}}$

$m = \frac{1}{\sqrt{6}}$

$n = \frac{1}{\sqrt{6}}$

DC's $= \langle l, m, n \rangle$, $\left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

$l = \cos\alpha$

$\cos\alpha = \frac{2}{\sqrt{6}}$

$\alpha = \cos^{-1}\frac{2}{\sqrt{6}}$

$\beta = \cos^{-1} m$

$\beta = \cos^{-1}\frac{1}{\sqrt{6}}$

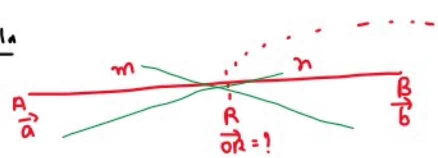
$\lambda = \cos^{-1}\frac{1}{\sqrt{6}}$

$l^2 + m^2 + n^2 = 1$

$\left(\frac{2}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2$

$\frac{4}{6} + \frac{1}{6} + \frac{1}{6} = \underline{\underline{1}}$

Section formula



If R is the Mid-Point

$$\overrightarrow{OR} = \frac{1\vec{b} + 1\vec{a}}{1+1}$$

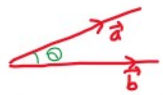
$\vec{OR} = \dfrac{1\vec{b} + 1\vec{a}}{1+1}$

$\boxed{\vec{OR} = \dfrac{\vec{a}+\vec{b}}{2}}$

Internally $\quad \vec{OR} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

externally $\quad \vec{OR} = \dfrac{m\vec{b} - n\vec{a}}{m+n}$

Dot Product of Two vectors
OR
Scalar product



Dot product denoted by $\vec{a}\cdot\vec{b}$

① $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\theta \qquad \boxed{0 \le \theta \le \pi}$

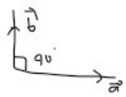
$\theta \in [0, \pi]$

② If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then θ is not defined.

Properties

(i) If $\vec{a} \perp \vec{b}$

$\theta = 90°$



$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos 90°$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot 0$

$\boxed{\vec{a}\cdot\vec{b} = 0}$

(ii) $\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cdot\cos 0°$

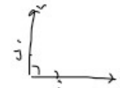
$\vec{a}\cdot\vec{a} = |\vec{a}|^2 \cdot 1$

$\boxed{\vec{a}\cdot\vec{a} = |\vec{a}|^2}$

(iii) $\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{i} = 0$



$\hat{i}\cdot\hat{i} = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

Ex $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a}\cdot\vec{b} = 2 - 6 - 2$

$= -6$

Real No. or Scalar

$\vec{a}\cdot\vec{b}$ = Real No. s.g.

(iv) If $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a}\cdot\vec{b} = (a\hat{i} + b\hat{j} + c\hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k})$

$= ax\,\hat{i}\cdot\hat{i} + ay\,\hat{i}\cdot\hat{j} + az\,\hat{i}\cdot\hat{k} + bx\,\hat{j}\cdot\hat{i} + by\,\hat{j}\cdot\hat{j} + bz\,\hat{j}\cdot\hat{k}$
$+ cx\,\hat{k}\cdot\hat{i} + cy\,\hat{k}\cdot\hat{j} + cz\,\hat{k}\cdot\hat{k}$

$= \boxed{ax + by + cz}$

Sol. 8 $\quad |\vec{a}| = ?$

$|\vec{b}| = ?$

$|\vec{a}| = |\vec{b}|$

$\theta = 60°$

$\vec{a}\cdot\vec{b} = \frac{1}{2}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\theta$

$\frac{1}{2} = |\vec{a}||\vec{a}|\cdot\cos 60°$

$\frac{1}{2} = |\vec{a}|^2\cdot\frac{1}{2}$

$1 = |\vec{a}|^2$

$|\vec{a}| = \sqrt{1}$

$\boxed{|\vec{a}| = 1}$

$|\vec{a}| = |\vec{b}| = 1$

① $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

② $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$

③ $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$

④ $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

Ex If $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $\mu = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$
and $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 2$

find $\mu = ?$ $\qquad -\frac{29}{2}$

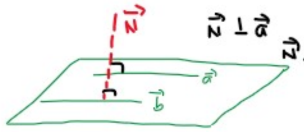
$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$0 = 9 + 16 + 4 + 2\mu$

$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\underline{(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})}$

$0 = 9 + 16 + 4 + 2\mu$

$\frac{-29}{2} = \mu$



$\vec{N} \perp \vec{a}$

$\vec{N} \perp \vec{b}$

Cross Product (vector product)

① $\vec{a}\times\vec{b} = |\vec{a}|\cdot|\vec{b}|\sin\theta\cdot\hat{n}$

$\hat{n}$ = unit vector of $\vec{N}$

② $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\cdot\sin\theta\cdot|\hat{n}|$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\cdot\sin\theta$

③ $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= \hat{i}(3-1) - \hat{j}(-2-1) + \hat{k}(2+3)$

$= 2\hat{i} + 3\hat{j} + 5\hat{k} \rightarrow \vec{N}$

④ $\vec{a}\times\vec{b}$ = vector qu.

$\hat{i}\times\hat{i} = |\hat{i}|\cdot|\hat{i}|\cdot\sin\theta$

④ $\hat{i}\times\hat{i} = 0$

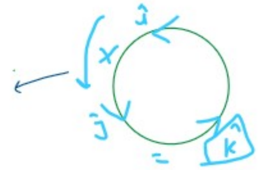
$\hat{j}\times\hat{j} = 0$

$\hat{k}\times\hat{k} = 0$

⑤ $\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$



⑥ $\hat{j}\times\hat{i} = -\hat{k}$

$\hat{k}\times\hat{j} = -\hat{i}$

$\hat{i}\times\hat{k} = -\hat{j}$

$\vec{N} \perp \vec{a}$

$\vec{N}\cdot\vec{a} = 0$

$4 - 9 + 5$

$\underline{\underline{0}}$

$\vec{N} \perp \vec{b}$

$2 + 3 - 5$

$\underline{\underline{0}}$

$\vec{c} = a + \vec{b} = 4\hat{i} + 4\hat{j} + 0\hat{k}$

$\vec{d} = a - \vec{b} = 2\hat{i} + 0\hat{k} + 4\hat{k}$

$\vec{c} \perp \quad \vec{d} \perp$

$\vec{a} \quad a \quad \vec{i}$

find projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \vec{a}\cdot\hat{b}$

projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \vec{b}\cdot\hat{a}$

$\vec{c}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(16) - 8\hat{k}$

$\vec{N} = 16\hat{i} - 16\hat{j} - 8\hat{k}$

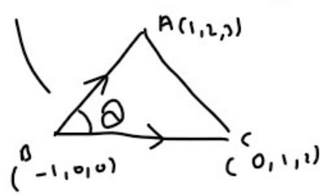
$\hat{N} = \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{256+256+64}}$

2 | 576
2 | 288
  | 144

256
256
64
576

$\hat{N} = \frac{16}{24}\hat{i} - \frac{16}{24}\hat{j} - \frac{8}{24}\hat{k}$

$= \frac{2\hat{i}}{3} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$



$\vec{BA} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{BC} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{BA}$ & $\vec{BC}$ Angle ?

let: $(\vec{BA}) = \vec{a}$

$\vec{BC} = \vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cdot\cos\theta$

$2 + 2 + 6 = \sqrt{4+4+9}\,\sqrt{1+1+4}\cdot\cos\theta$

$10 = \sqrt{17}\,\sqrt{6}\,\cos\theta$

2 | 102
  | 51

$\frac{9}{51}$

$\ldots + 4 + 9 \quad \sqrt{1+1+4} \cdot \cos\theta$

$10 = \sqrt{17}\sqrt{6}\cos\theta$

$10 = \sqrt{102}\cos\theta$

$\cos\theta = \frac{10}{\sqrt{102}} \qquad \theta = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$